



# The Thomson effect and the ideal equation on thermoelectric coolers



HoSung Lee\*

Mechanical and Aeronautical Engineering, Western Michigan University, 1903 W. Michigan Ave, Kalamazoo, MI 49008, USA

## ARTICLE INFO

### Article history:

Received 5 October 2012

Received in revised form

19 April 2013

Accepted 22 April 2013

Available online 22 May 2013

### Keywords:

Thermoelectrics

Thermoelectricity

Thermoelectric coolers

Thomson relations

Thomson effect

## ABSTRACT

The formulation of the classical basic equations for a thermoelectric cooler from the Thomson relations to the non-linear differential equation with Onsager's reciprocal relations was performed to basically study the Thomson effect in conjunction with the ideal equation. The ideal equation is obtained when the Thomson coefficient is assumed to be zero. The exact solutions derived for a commercial thermoelectric cooler module provided the temperature distributions including the Thomson effect. The positive Thomson coefficient led to a slight improvement on the performance of the thermoelectric device while the negative Thomson coefficient led to a slight decline of the performance. The comparison between the exact solutions and the ideal equation on the cooling power and the coefficient of performance over a wide range of temperature differences showed close agreement. In conclusion, the Thomson effect is small for typical commercial thermoelectric coolers and the ideal equation effectively predicts the performance.

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## 1. Introduction

Thermoelectric phenomena are useful and have drawn much attention since the discovery of the phenomena in the early nineteenth century. The barriers to applications were low efficiencies and the availability of materials. In 1821, Thomas J. Seebeck discovered that an electromotive force or potential difference could be produced by a circuit made from two dissimilar wires when one junction was heated. This is called the *Seebeck effect*. In 1834, Jean Peltier discovered the reverse process that the passage of an electric current through a thermocouple produces heating or cooling depended on its direction [1]. This is called the *Peltier effect* (or *Peltier cooling*). In 1854, William Thomson discovered that if a temperature difference exists between any two points of a current-carrying conductor, heat is either absorbed or liberated depending on the direction of current and material [2]. This is called the *Thomson effect* (or *Thomson heat*). These three effects are called the *thermoelectric effects*. Thomson also developed important relationships between the above three effects with the reciprocal relations of the kinetic coefficients (or sometimes the so-called symmetry of the kinetic coefficients) under a peculiar assumption that the thermodynamically reversible and

irreversible processes in thermoelectricity are separable [2]. The relationship developed is called the *Thomson (or Kelvin) relations*. The necessity for the assumption remained an objection to the theory until the advent of the new thermodynamics [3–6]. The relationship was later completely confirmed by experiments to be essentially a consequence of the Onsager's Reciprocal Principle in 1931 [3].

As supported by Onsager's Principle, the Thomson relations provide a simple expression for Peltier cooling, which is the product of the Seebeck coefficient, the temperature at the junction, and the current. This Peltier cooling is the principal thermoelectric cooling mechanism. There are two counteracting phenomena, which are the Joule heating and the thermal conduction. The net cooling power is the Peltier cooling minus these two effects. Actually the Joule heating affects the thermal conduction and consequently the Peltier cooling is subtracted only by the thermal conduction. Expressing the net cooling power in terms of the heat flux vector  $\vec{q}$  gives [7]

$$\vec{q} = \alpha T \vec{j} - k \nabla T \quad (1)$$

This is the most important formula, where  $\alpha$  is the Seebeck coefficient,  $T$  the absolute temperature,  $\vec{j}$  the current density and  $k$  the thermal conductivity. Equation (1) appears simple, but the interpretation was somewhat formidable because the  $\alpha T$  in the equation was obtained by conjecture and the reciprocal relations.

\* Tel.: +1 269 276 3429; fax: +1 269 276 3421.

E-mail address: [hosung.lee@wmich.edu](mailto:hosung.lee@wmich.edu).

| Nomenclature     |   |                      |  |
|------------------|---|----------------------|--|
| A                | cross-sectional area of thermoelement (m <sup>2</sup> )               | T                    | absolute temperature (K)   |
| COP              | the coefficient of performance, dimensionless                         | T <sub>c</sub>       | low junction temperature (K)   |
| c <sub>p</sub>   | specific heat capacity (J/kg K)                                       | T <sub>h</sub>       | high junction temperature (K)  |
| $\vec{E}$        | electric field vector (V/m)   | T <sub>1</sub>       | low junction temperature (K)   |
| I                | electric current (A)  | T <sub>2</sub>       | high junction temperature (K)  |
| $\vec{j}$        | electric current density vector (A/m <sup>2</sup> )                   | ΔT                   | temperature difference (K), T <sub>2</sub> – T <sub>1</sub>            |
| j                | electric current density (A/m <sup>2</sup> )                          | V                    | volume (m <sup>3</sup> )   |
| J <sub>i</sub>   | arbitrary quantities  | $\dot{W}$            | work per unit time $\dot{W} = \alpha I \Delta T + I^2 R$               |
| K                | thermal conductance (W/K)   | x                    | distance of thermoelement leg (m)                                      |
| L                | length of thermoelement (m)   | x <sub>i</sub>       | arbitrary quantities   |
| k                | thermal conductivity (W/m K)  | X <sub>i</sub>       | arbitrary quantities: $\partial x_i / \partial t$                      |
| $\vec{q}$        | heat flux vector (W/m <sup>2</sup> )                                  | <b>Greek symbols</b> |  |
| $\dot{q}$        | heat generation (W/m <sup>3</sup> )                                   | α                    | Seebeck coefficient (V/K)  |
| $\dot{Q}_c$      | cooling power (W)   | β                    | dimensionless number $\beta = IT_2(d\alpha/dT)/(Ak/L)$                 |
| $\dot{Q}_1$      | cooling power (W)   | γ                    | dimensionless number $\gamma = I^2 R / (Ak \Delta T / L)$              |
| R                | electrical resistivity (Ω)  | φ                    | the ratio of ΔT to T <sub>2</sub>                                      |
| R <sub>ij</sub>  | kinetic coefficients  | ρ                    | density (kg/m <sup>3</sup> )   |
| S <sub>gen</sub> | entropy generation per unit volume (J/m <sup>3</sup> K)               | ρ                    | electric resistivity (Ω m)   |
| $\dot{S}_{gen}$  | entropy generation per unit time and unit volume (J/m <sup>3</sup> K) | τ                    | Thomson coefficient (V/K)  |
| S <sub>gen</sub> | entropy generation (J/K)  | θ                    | dimensionless temperature $\theta = (T - T_1)/(T_2 - T_1)$             |
| $\dot{S}_{gen}$  | entropy generation per unit time (W/K)                                | ξ                    | dimensionless distance $\xi = x/L$                                     |
|                  |   | Θ <sub>1</sub>       | dimensionless cooling power $\Theta_1 = \dot{Q}_1 / (Ak \Delta T / L)$ |
|                  |   | Π                    | dimensionless Peltier cooling $\Pi = \alpha(T_1)I / (Ak/L)$            |
|                  |   | Ψ                    | dimensionless work $\Psi = \dot{W} / (Ak \Delta T / L)$                |

In fact, the three thermoelectric effects that are known to be reversible are effectively combined into the  $\alpha T$  [8–10]. The second term gives the thermal conduction that is obviously irreversible. A steady-state heat diffusion equation is given by [7]

$$\vec{\nabla} \cdot (k \vec{\nabla} T) + j^2 \rho - T \frac{d\alpha}{dT} \vec{j} \cdot \vec{\nabla} T = 0 \quad (2)$$

where  $\rho$  is the electrical resistivity. Equation (2) is a non-linear differential equation. The Thomson coefficient is given by

$$\tau = T \frac{d\alpha}{dT} \quad (3)$$

In order to obtain the net cooling power, Equation (2) is first solved for the thermal conduction including the Joule heating and then Equation (1) can give the cooling power at the junction. It is often assumed [8–10,16,17] that the Thomson effect was absent or negligible or, in other words,  $\alpha$  was independent of temperature. In such a case, Equation (2) becomes an ordinary differential equation and can be easily solved for the thermal conduction and then Equation (1) yields a well-known equation as

$$\dot{Q}_c = \alpha T_c I - \frac{1}{2} I^2 R - K(T_h - T_c) \quad (4)$$

where  $\dot{Q}_c$  is the cooling power,  $I$  the current,  $T_c$  the low junction temperature,  $T_h$  the high junction temperature,  $R$  the electrical resistance, and  $K$  the thermal conductance. Equation (4) is called the ideal equation and has been widely used in science and industry [8,10,15,16] often with a fair agreement with the experiments [9,13].

The third term in Equation (2) was regarded as the Thomson effect. The influence of the Thomson effect was discussed in the literature [11–14] by numerically solving Equation (2), in which the Seebeck coefficient  $\alpha$  was dependent of temperature. The

exact solutions of Equation (2) assuming that the Thomson coefficient  $\tau$  is constant indicated that the Thomson effect would improve the performance significantly [12,13]. More realistic work on the Thomson coefficient assuming that  $d\alpha/dT$  is a linear function of temperature was numerically conducted using a finite difference method and also experimentally compared with a commercial module showing slight improvement on the performance [13]. Numerical simulations on the Thomson effect using CFD software ANSYS for miniature thermoelectric coolers revealed that the cooling power can be improved by a factor of 5–7% [14] by including the Thomson effect. There is an experimental work [18] showing that the Thomson effect is responsible for reduction of the figure of merit on a thermoelectric generator with an increase of temperature difference, whereas the researchers anticipated enhancement of the figure of merit on a thermoelectric cooler. There were attempts [19,20] to improve the performance of thermoelectric devices by optimizing the temperature dependence of the material properties wherein the Thomson effect was taken into account although the improvement could not be demonstrated by the measurements. There is a rigorous numerical analysis [21] for emerging materials and large temperature gradients in thermoelectric generators how well the ideal equation predicts the exact solutions including the Thomson effect. It is analytically proven [21] that the ideal equation produces the exact solution output power and efficiency despite its limiting assumptions if an integral-averaged Seebeck coefficient is used.

From the review of the above experimental and theoretical work, the present author feels that further realistic studies are required to clarify the Thomson effect related to the ideal equation in temperature distributions, particularly in the positive and negative Thomson coefficients. The present work studies the Thomson effect in conjunction with the ideal equation from the formulation of the basic equations to the numerical simulations in light of a real commercial thermoelectric module.

## 2. Formulation of basic equations

### 2.1. Onsager's reciprocal relations

The second law of thermodynamics with no mass transfer ( $\Delta S = 0$ ) in an isotropic substance provides an expression for the entropy generation.

$$S_{\text{gen}} = - \int_1^2 \frac{\delta Q}{T} > 0 \quad (5)$$

Suppose that the derivatives of the entropy generation  $s_{\text{gen}}$  per unit volume with respect to arbitrary quantities  $x_i$  are quantities  $J_i$  [4,7] as

$$-\frac{\partial s_{\text{gen}}}{\partial x_i} = J_i \quad (6)$$

At maximum  $s_{\text{gen}}$ ,  $J_i$  is zero. Accordingly, at the state close to equilibrium (or maximum),  $J_i$  is small [4]. The entropy generation per unit time and unit volume is

$$-\dot{s}_{\text{gen}} = -\frac{\partial s_{\text{gen}}}{\partial t} = \frac{\partial x_i}{\partial t} J_i \quad (7)$$

We want to express  $\partial x_i / \partial t$  as  $X_i$ . Then, we have

$$-\dot{s}_{\text{gen}} = X_i J_i \quad (8)$$

The  $X_i$  are usually a function of  $J_i$ . Onsager [3] stated that if  $J_i$  were completely independent, we should have the relation, expanding  $X_i$  in the powers of  $J_i$  and taking only linear terms [7]. The smallness of  $J_i$  in practice allows the linear relations being sufficient.

$$X_i = \sum_j R_{ij} J_j \quad (9)$$

where  $R_{ij}$  are called the kinetic coefficients. Hence, we have

$$-\dot{s}_{\text{gen}} = \sum_i X_i J_i \quad (10)$$

It is necessary to choose the quantities  $X_i$  in some manner, and then to determine the  $J_i$ . The  $X_i$  and  $J_i$  are conveniently determined simply by means of the formula for the rate of change of the total entropy generation.

$$\dot{s}_{\text{gen}} = - \int \sum_i X_i J_i dV > 0 \quad (11)$$

For two terms, we have

$$\dot{s}_{\text{gen}} = - \int [X_1 J_1 + X_2 J_2] dV > 0 \quad (12)$$

And

$$X_1 = R_{11} J_1 + R_{12} J_2 \quad (13)$$

$$X_2 = R_{21} J_1 + R_{22} J_2 \quad (14)$$

From these equations, one can assert that the kinetic coefficients are symmetrical with respect to the suffixes 1 and 2.

$$R_{12} = R_{21} \quad (15)$$

which is called the reciprocal relations.

### 2.2. Basic equations

Let us consider a non-uniformly heated thermoelectric material. For an isotropic substance, the continuity equation for a constant current gives

$$\vec{\nabla} \cdot \vec{j} = 0 \quad (16)$$

The electric field  $\vec{E}$  is affected by the current density  $\vec{j}$  and the temperature gradient  $\vec{\nabla} T$ . The coefficients are known according to the Ohm's law and the Seebeck effect [7]. The field is then expressed as

$$\vec{E} = \vec{j} \rho + \alpha \vec{\nabla} T \quad (17)$$

where  $\rho$  is the electrical resistivity and  $\alpha$  is the Seebeck coefficient. Solving for the current density gives

$$\vec{j} = \frac{1}{\rho} (\vec{E} - \alpha \vec{\nabla} T) \quad (18)$$

The heat flux  $\vec{q}$  is also affected by the both field  $\vec{E}$  and temperature gradient  $\vec{\nabla} T$ . However, the coefficients were not readily attainable at that time. Thomson in 1854 arrived at the relationship assuming that thermoelectric phenomena and thermal conduction are independent. Later, Onsager [3] supported the relationship by presenting the reciprocal principle which was experimentally proved but failed the phenomenological proof [6]. Here we derive the Thomson relationship using Onsager's principle. The heat flux with arbitrary coefficients is

$$\vec{q} = C_1 \vec{E} - C_2 \vec{\nabla} T \quad (19)$$

The general heat diffusion equation is

$$-\vec{\nabla} \cdot \vec{q} + \dot{q} = \rho c_p \frac{\partial T}{\partial t} \quad (20)$$

For steady state, we have

$$-\vec{\nabla} \cdot \vec{q} + \dot{q} = 0 \quad (21)$$

where  $\dot{q}$  is expressed by [7]

$$\dot{q} = \vec{E} \cdot \vec{j} = j^2 \rho + \vec{j} \cdot \alpha \vec{\nabla} T \quad (22)$$

Expressing Equation (5) with Equation (21) in a manner that the heat flux gradient  $\vec{\nabla} \cdot \vec{q}$  has a minus sign, the entropy generation per unit time will be

$$\dot{s}_{\text{gen}} = - \int_1^2 \frac{\delta Q}{T} = \int \frac{1}{T} (-\vec{\nabla} \cdot \vec{q} + \dot{q}) dV \quad (23)$$

Using Equation (22), we have

$$\dot{s}_{\text{gen}} = \int \frac{\vec{E} \cdot \vec{j}}{T} - \int \frac{\vec{\nabla} \cdot \vec{q}}{T} dV \quad (24)$$

The second term is integrated by parts, using the divergence theorem and noting that the fully transported heat does not produce entropy since the volume term cancels out the surface contribution [7]. Then, we have

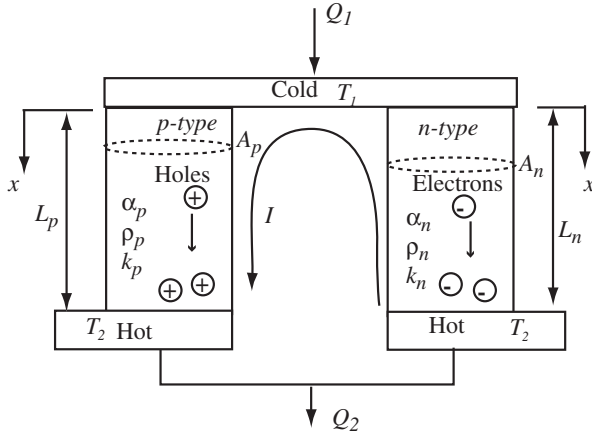


Fig. 1. Thermoelectric cooler with p-type and n-type thermoelements.

$$\dot{S}_{\text{gen}} = - \int \left( -\frac{\vec{E} \cdot \vec{j}}{T} + \frac{\vec{q} \cdot \vec{\nabla} T}{T^2} \right) dV \quad (25)$$

If we take  $\vec{j}$  and  $\vec{q}$  as  $X_1$  and  $X_2$ , then the corresponding quantities  $J_1$  and  $J_2$  are the components of the vectors  $-\vec{E}/T$  and  $\vec{\nabla} T/T^2$  [7]. Accordingly, as shown in Equations (13) and (14) with Equation (19), we have

$$\vec{j} = -\frac{1}{\rho} T \left( -\frac{\vec{E}}{T} \right) - \frac{1}{\rho} \alpha T^2 \frac{\vec{\nabla} T}{T^2} \quad (26)$$

$$\vec{q} = -C_1 T \left( -\frac{\vec{E}}{T} \right) - C_2 T^2 \frac{\vec{\nabla} T}{T^2} \quad (27)$$

where the reciprocal relations in Equation (15) gives

$$C_1 T = \frac{1}{\rho} \alpha T^2 \quad (28)$$

Thus, we have  $C_1 = \alpha T / \rho$ . Inserting this and Equation (17) into (27) gives

$$\vec{q} = \alpha T \vec{j} - \left( C_2 - \frac{\alpha^2 T}{\rho} \right) \vec{\nabla} T \quad (29)$$

We introduce the Wiedemann–Franz law that is  $L_0 = k\rho/T$ , where  $L_0$  is the Lorentz number (constant). Expressing  $k$  as  $L_0 T / \rho$ , we find that the coefficient  $(C_2 - \alpha^2 T / \rho)$  is nothing more than the thermal conductivity  $k$ . Finally, the heat flow density vector (heat flux) is expressed as

$$\vec{q} = \alpha T \vec{j} - k \vec{\nabla} T \quad (30)$$

This is the most essential equation for thermoelectric phenomena. The second term pertains to the thermal conduction and the term of interest is the first term, which gives the thermoelectric effects: directly the Peltier cooling but indirectly the Seebeck effect and the Thomson heat [8–10]. Inserting Equations (17) and (30) into Equation (25) gives

$$\dot{S}_{\text{gen}} = \int \left( \frac{j^2 \rho}{T} + \frac{k (\vec{\nabla} T)^2}{T^2} \right) dV > 0 \quad (31)$$

The entropy generation per unit time for the irreversible processes is indeed greater than zero since  $\rho$  and  $k$  are positive.

Note that the Joule heating and the thermal conduction in the equation are indeed irreversible. This equation satisfies the requirement for Onsager's reciprocal relations as shown in Equation (12). Substituting Equations (22) and (30) in Equation (21) yields

$$\vec{\nabla} \cdot (k \vec{\nabla} T) + j^2 \rho - T \frac{d\alpha}{dT} \vec{j} \cdot \vec{\nabla} T = 0 \quad (32)$$

The Thomson coefficient  $\tau$ , originally obtained from the Thomson relations, is written

$$\tau = T \frac{d\alpha}{dT} \quad (33)$$

In Equation (32), the first term is the thermal conduction, the second term is the Joule heating, and the third term is the Thomson heat. Note that if the Seebeck coefficient  $\alpha$  is independent of temperature, the Thomson coefficient  $\tau$  is zero and then the Thomson heat is absent.

### 2.3. Exact solutions

Let us consider one of p- and n-type thermoelements as shown in Fig. 1, knowing that p- and n-type thermoelements produce the same results if the materials and dimensions are assumed to be similar. Equation (32) for one dimensional analysis at steady state gives the non-linear differential governing equation as

$$\frac{d^2 T}{dx^2} - \frac{I \frac{d\alpha}{dT}}{Ak} T \frac{dT}{dx} + \frac{I^2 \rho}{A^2 k} = 0 \quad (34)$$

where  $A$  is the cross-sectional area of the thermoelement. We want to make it dimensionless. The boundary conditions will be  $T(0) = T_1$  and  $T(L) = T_2$ . Then, Let

$$\theta = \frac{T - T_1}{T_2 - T_1} \text{ and } \xi = \frac{x}{L} \quad (35)$$

where  $L$  is the element length. Then, the dimensionless differential equation is

$$\frac{d^2 \theta}{d\xi^2} - \beta ((\theta - 1)\phi + 1) \frac{d\theta}{d\xi} + \gamma = 0 \quad (36)$$

where the dimensionless numbers are defined as follows. The boundary condition will be that  $\theta(0) = 0$  and  $\theta(1) = 1$ . Equation (36) was formulated on the basis of the high junction temperature  $T_2$  to meet the commercial products.

$$\beta = \frac{\pi_2 \frac{d\alpha}{dT} \Delta T}{Ak \frac{\Delta T}{L}} \quad (37)$$

where  $\beta$  is the ratio of the Thomson heat to the thermal conduction. Note that  $\beta$  is not a function of  $\Delta T$ .

$$\gamma = \frac{I^2 R}{Ak \frac{\Delta T}{L}} \quad (38)$$

where  $\gamma$  is the ratio of the Joule heating to the thermal conduction.

$$\phi = \frac{\Delta T}{T_2} \quad (39)$$

where  $\phi$  is the ratio of temperature difference to the high junction temperature. The temperature difference is

$$\Delta T = T_2 - T_1 \quad (40)$$

where  $\Delta T$  is the high junction temperature  $T_2$  minus the low junction temperature  $T_1$ . Therefore,  $T_1$  is a function of  $\Delta T$  since  $T_2$  is fixed. The cooling power at the cold junction using Equation (30) is

$$\dot{Q}_1 = \alpha(T_1)T_1I + \left(-kA\frac{dT}{dx}\right)_{x=0} \quad (41)$$

where the first term is the Peltier cooling and the second term is the thermal conduction. It has been customary in the literature for the exact solution wherein the Seebeck coefficient  $\alpha$  is evaluated at the cold junction temperature  $T_1$ . The dimensionless cooling power is expressed

$$\Theta_1 = \Pi \left( \frac{1-\phi}{\phi} \right) - \frac{d\theta}{d\xi} \bigg|_{\xi=0} \quad (42)$$

where  $\Theta_1$  is the dimensionless cooling power, which is

$$\Theta_1 = \frac{\dot{Q}_1}{Ak\Delta T/L} \quad (43)$$

and the dimensionless Peltier cooling is

$$\Pi = \frac{\alpha(T_1)I}{Ak/L} \quad (44)$$

The work per unit time gives  $\dot{W} = \alpha I \Delta T + I^2 R$  [10]. Then the dimensionless work per unit time  $\Psi$  is expressed by

$$\Psi = \Pi + \gamma \quad (45)$$

where  $\Psi = \dot{W}/(Ak\Delta T/L)$ . Then, the coefficient of performance (COP) for the thermoelectric cooler is determined as the cooling power over the work as

$$\text{COP} = \frac{\Theta_1}{\Psi} \quad (46)$$

Equation (36) can be exactly solved for the temperature distributions conveniently with mathematical software Mathcad and then the cooling power of Equation (41) can be obtained, which are the exact solutions including the Thomson effect.

#### 2.4. The ideal equation

According to the assumption made by the both Thomson relations and Onsager's reciprocal relations, the thermoelectric effects and the thermal conduction in Equation (30) are completely independent, which implies that each term may be separately dealt with. In fact, separately dealing with the reversible processes of the three thermoelectric effects yielded the Peltier cooling of the  $\alpha TI$  in the equation [8–10]. However, under the assumption that the Thomson coefficient is negligible or the Seebeck coefficient is independent of temperature, Equation (34) easily provides the exact solution of the temperature distribution. Then, Equation (41) with the temperature distribution for the cooling power at the cold junction reduces to

$$\dot{Q}_1 = \alpha(T_{\text{avg}})T_1I - \frac{1}{2}I^2R - \frac{Ak}{L}(T_2 - T_1) \quad (47)$$

which appears simple but is a robust equation with a usually good agreement with experiments and with comprehensive applications in science and industry. This is here called the *ideal equation*. The ideal equation assumes that the Seebeck coefficient  $\alpha$  be evaluated

at the average temperature of  $\Delta T$  [15,16] as a result of the internal phenomena being imposed on the surface phenomena.

### 3. Results and discussion

In order to realistically study the Thomson effect on the temperature distributions with the temperature dependent Seebeck coefficient, a real commercial module of Laird CP10-127-05 was chosen. Both the temperature dependent properties and the dimensions for the module were provided by the manufacturer, which are shown in Table 1 and the temperature dependent Seebeck coefficient is graphed in Fig. 2.

To reveal the Thomson effect, only the Seebeck coefficients are considered to be dependent of the temperature while the electrical resistivity and the thermal conductivity are constant as shown in Table 1. In Fig. 2, the Seebeck coefficient increases with increasing the temperature up to about 350 K and then decreases with increasing the temperature, so that there is a peak value. For the feasible solution with the non-linear differential equation, the linear curve fits for the Seebeck coefficient were used in the present work. In Fig. 2, the linear curve fit of the first part for a range from 250 K to 350 K was used in the present work, while the other linear curve fit of the second part for the range from 350 K to 450 K was used separately. The linear curve fits of the two parts are shown in Table 1. Some reports [11–14] illustrated used only the monotonically increasing Seebeck coefficients. Therefore, the negative slope of the Seebeck coefficient was not presented. As mentioned previously, the absolute values of the temperature dependent Seebeck coefficients for p- and n-type elements are assumed to be the same, but the sign of n-type element's coefficient is negative while the sign of p-type element's coefficient is positive. In reality, the Seebeck coefficients in the p- and n-type elements are different as the absolute values vary with temperature. Fig. 2 would be the arithmetic average of the two. By doping technique (intentionally introducing impurities into a pure semiconductor), n-type semiconductor carries predominantly free electrons moving to the opposite direction of current applied (which causes the negative sign of the Seebeck coefficient), while p-type semiconductor carries free holes in the same manner but moving to the same direction of current applied (which causes the positive sign). Likewise, p- and n-type thermoelements can be arranged electrically in series and thermally in parallel to form a module. The number of electrons and holes can be controlled through the introduction of impurities (doping), which are called donors or acceptors.

The dimensionless differential equation, Equation (36), was developed on the basis of a fixed high junction temperature  $T_2$  so that the low junction temperature  $T_1$  decreases as  $\Delta T$  increases.  $\beta$  in Equation (37) is the dimensionless number indicating approximately the ratio of the Thomson heat to the thermal conduction.  $\beta$  is a function only of the slope  $d\alpha/dT$  and the current  $I$ . Equation (36)

**Table 1**  
Maximum values, dimensions, and the properties for a commercial module.

| TEC Module (Laird CP10-127-05)               | At 25 °C  |
|--|---|
| $\Delta T_{\text{max}}$ (°C)                 | 67  |
| $I_{\text{max}}$ (A)                         | 3.9   |
| $Q_{\text{max}}$ (W)                         | 34.3  |
| $L$ (mm), element length                     | 1.253   |
| $A$ (mm <sup>2</sup> ), cross-sectional area | 1   |
| # of thermocouples                           | 127   |
| $\alpha$ (T), ( $\mu\text{V/K}$ )            | $0.2068T + 138.78$ , $250 \text{ K} < T < 350 \text{ K}$  |
| $\alpha$ (T), ( $\mu\text{V/K}$ )            | $-0.2144T + 281.02$ , $350 \text{ K} < T < 450 \text{ K}$ |
| $\rho$ ( $\Omega \text{ cm}$ ) at 27 °C      | $1.01 \times 10^{-3}$                                     |
| $k$ (W/cmK) at 27 °C                         | $1.51 \times 10^{-2}$                                     |
| Module size                                  | $30 \times 30 \times 3.2 \text{ mm}$                      |



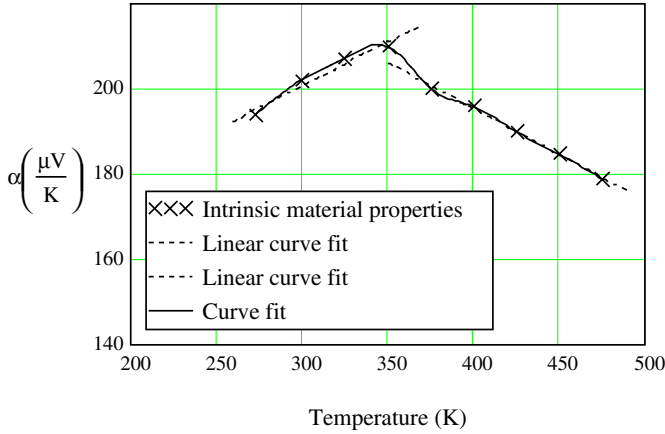


Fig. 2. Seebeck coefficient as a function of temperature for Laird products.

with the given boundary conditions was solved for the temperature distributions conveniently using mathematical software Mathcad.

A typical value of  $\beta = 0.2$  was obtained for the commercial product at  $T_2 = 298$  K and  $I_{\max} = 3.9$  A. And a hypothetical value of 1.0 that is fivefold larger than the typical value of 0.2 was used in order to closely examine the Thomson effect.  $\gamma$  in Equation (38) is the dimensionless number indicating approximately the ratio of the Joule heating to the thermal conduction.

Fig. 3(a) and (b) show the temperature distributions at  $T_2 = 298$  K and  $I = 3.9$  A as a function of  $\beta$  for  $\Delta T = 10$  K ( $\gamma = 15.97$ )

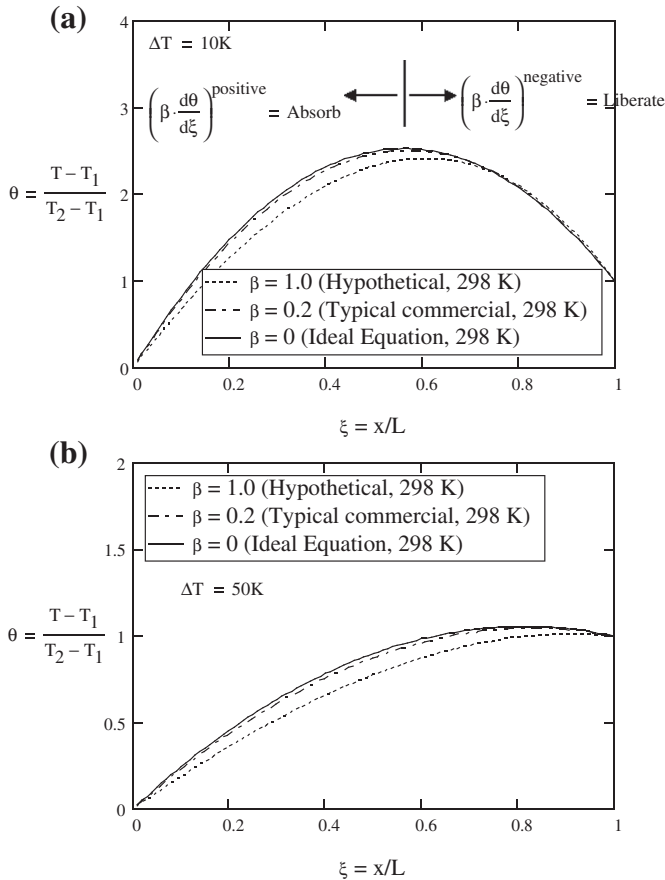
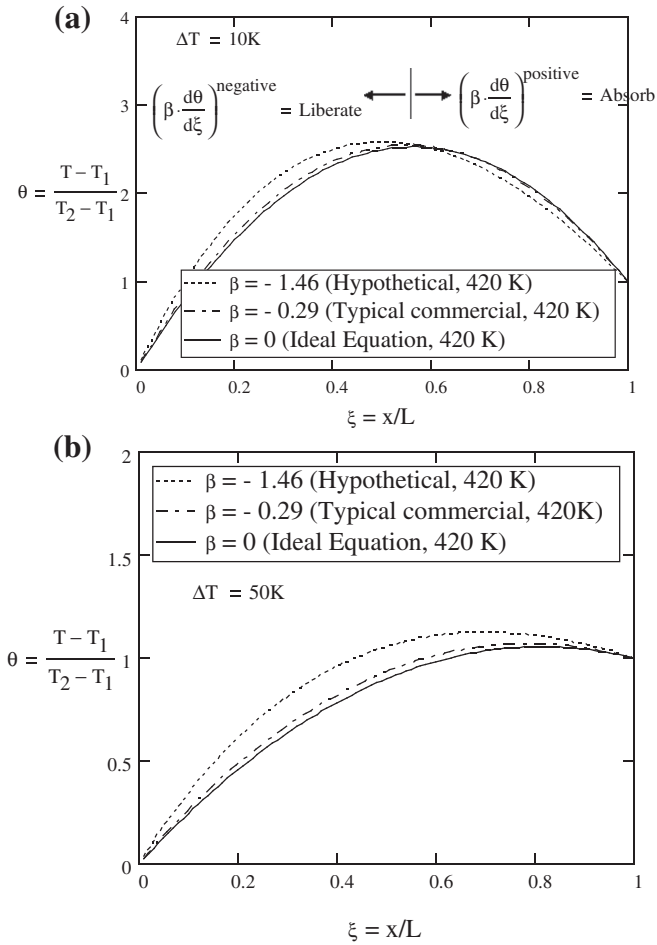


Fig. 3. (a)  $\Delta T = 10$  K ( $\gamma = 15.97$ ), and (b)  $\Delta T = 50$  K ( $\gamma = 3.19$ ), dimensionless temperature  $\theta$  vs. dimensionless distance  $\xi$  as a function of  $\beta$ . The operating conditions are  $T_2 = 298$  K and  $I = 3.9$  A.

and  $\Delta T = 50$  K ( $\gamma = 3.19$ ). In Fig. 3(a), the Joule heating appears dominant with a high value of  $\gamma = 15.97$ , while in Fig. 3(b) the thermal conduction is approximately balanced with the Joule heating with a low value of  $\gamma = 3.19$ . For the commercial cases in the both figures with  $\beta = 0.2$ , the Thomson effects appear small compared to the ideal equation at  $\beta = 0$ . The hypothetical cases at  $\beta = 1.0$  obviously provide salient examples for the Thomson effect. Since the temperature gradient at  $\xi = 0$  counteracts the Peltier cooling, the slightly lower temperature distribution near zero acts as improving the net cooling power. The Thomson heat is effectively described by the product of  $\beta$  and  $d\theta/d\xi$ . The Thomson heat acts as absorbing heat when the  $\beta d\theta/d\xi$  is positive, while it acts as liberating heat when the  $\beta d\theta/d\xi$  is negative, which are shown in Fig. 3(a). In the first half of the temperature distribution, both  $\beta$  and  $d\theta/d\xi$  are positive, so that the product is positive. It is known that the moving charged electrons or holes (see Fig. 1) transport not only the electric energy but also the thermal energy as absorbing and liberating depending on the sign of  $\beta d\theta/d\xi$  along the thermoelement. The temperature distribution is slightly shifted to the right giving the lower temperature distribution near  $\xi = 0$ , which results in the improved cooling power at the cold junction. It is interesting to note that the product of the current  $I$  and the Seebeck coefficient  $\alpha$  in Equation (37) is always positive, therefore making no contribution to the final sign of the Thomson heat. If the current is positive in a p-type element, the sign of  $\alpha$  is positive. If the current is negative in an n-type element, the sign of  $\alpha$  is negative (see Fig. 1). Therefore, the product is always positive.

Now consider the linear curve fit of the second part (for the range from 350 K to 450 K) of the temperature-dependent Seebeck coefficient in Fig. 2, where the  $\beta$  in Equation (37) is negative. Fig. 4(a) and (b) show the temperature distributions at  $T_2 = 420$  K and  $I = 3.9$  A as a function of  $\beta$  for  $\Delta T = 10$  K ( $\gamma = 15.97$ ) and  $\Delta T = 50$  K ( $\gamma = 3.19$ ). For the commercial cases at  $\beta = -0.29$  in the both figures, the Thomson effect again appears small compared to the ideal equation at  $\beta = 0$ . The hypothetical cases at  $\beta = -1.46$  obviously provide salient examples for the Thomson effect. Note that the temperature distribution is slightly shifted to the left giving the higher temperature gradient near  $\xi = 0$ , which results in the reduced cooling power at the cold junction. This is opposite of the cases at  $T_2 = 298$  K shown in Fig. 3(a) and (b). In Fig. 4(a), it is seen that the first and second half segments of the thermoelement exhibit the heat liberation and absorption, while the moving charged carriers (electrons or holes) are moving from left to right. Actually the moving carriers are resultantly at a speed from left to right, while numerous carriers are still moving in random directions, which can transport the thermal energy from right to left without problems.

Equation (41) represents the net cooling power which is calculated by subtracting the thermal conduction from the Peltier cooling. Each term of the equation is separately presented in Fig. 5(a) and (b) in order to examine the fraction of the thermal conduction to the Peltier cooling as well as the Thomson effect in terms of the heat transfer rate. In Fig. 5(a) for  $\Delta T = 10$  K, the percentage of the thermal conduction to the Peltier cooling at  $I = 3.9$  A is 46.2%, while, in Fig. 5(b) for  $\Delta T = 50$  K, the percentage is 81.6%. This indicates that the significant portion of heat generated by the Peltier cooling is wasted surprisingly by the thermal conduction and this waste robustly increases as  $\Delta T$  increases. Note that the net cooling power is zero at  $I = 1.5$  A. No gain in the cooling power is seen until after  $I = 1.5$  A. It would be desirable from the above cognition to develop a thermoelectric material that has a low thermal conductivity to minimize the waste heat by the thermal conduction. However, lowering the thermal conductivity without affecting the electrical conductivity is challenging according to the Wiedemann–Franz law.



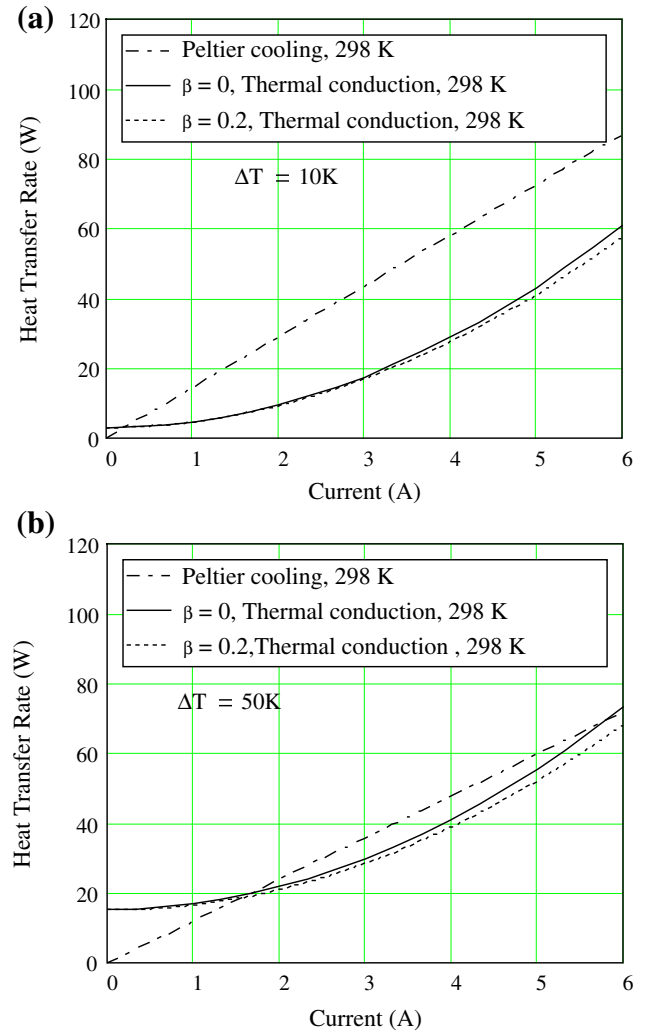
**Fig. 4.** (a)  $\Delta T = 10$  K ( $\gamma = 15.97$ ), and (b)  $\Delta T = 50$  K ( $\gamma = 3.19$ ), dimensionless temperature  $\theta$  vs. dimensionless distance  $\xi$  as a function of  $\beta$ . The operating conditions are  $T_2 = 420$  K and  $I = 3.9$  A.

The net cooling power  $\dot{Q}_1$  in Equation (41) and the coefficient of performance (COP) in Equation (46) along the current at both  $T_2 = 298$  K (thin lines) and  $420$  K (thick lines) for  $\Delta T = 10$  K are presented in Fig. 6. The maximum current for this commercial module is  $3.9$  A as shown in Table 1, so the operating current barely exceeds the maximum current in practice. The slight improvement by the Thomson effect on  $\dot{Q}_1$  at  $298$  K is also reflected on the corresponding COP wherein the effect appears even smaller. Of interest is that  $\dot{Q}_1$  at  $T_2 = 420$  K makes in the opposite way. The Thomson effect improves  $\dot{Q}_1$  at  $298$  K while it deteriorates  $\dot{Q}_1$  at  $420$  K.

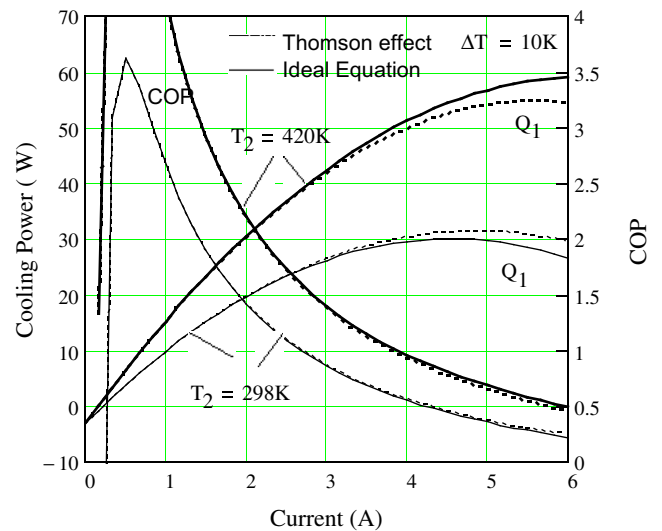
As  $\Delta T$  increases, the both cooling power and COP decrease, which is shown in Fig. 7. There is no appreciable change on the Thomson effect with increasing  $\Delta T$ .

The net cooling powers at  $T_2 = 298$  K and  $420$  K as a function of  $\Delta T$  are presented in Fig. 8. At a glance the exact solutions ( $\beta \neq 0$ ) and the ideal equation ( $\beta = 0$ ) are very close to each other. Furthermore, the two at  $298$  K are in good agreement with the available data of commercial products which are usually empirical data. It should be pointed out that the gaps between the Thomson effect and the ideal equation at both  $T_2 = 298$  K and  $420$  K remain unchanged, which is interpreted to be good for the predictability by the ideal equation. It is postulated that the ideal equation is very effective for prediction of the performance of thermoelectric coolers, if the averaged Seebeck coefficient is used.

In order to elucidate the agreement between the exact solutions, the ideal equation, and the numerical simulations, the numerical



**Fig. 5.** (a)  $\Delta T = 10$  K ( $\gamma = 15.97$ ), and (b)  $\Delta T = 50$  K ( $\gamma = 3.19$ ), heat transfer rate vs. current at  $T_2 = 298$  K.



**Fig. 6.** Cooling power and COP vs. current at  $T_2 = 298$  K and  $420$  K for  $\Delta T = 10$  K  $\beta = -0.29$  at  $T_2 = 420$  K and  $\beta = 0.2$  at  $T_2 = 298$  K.

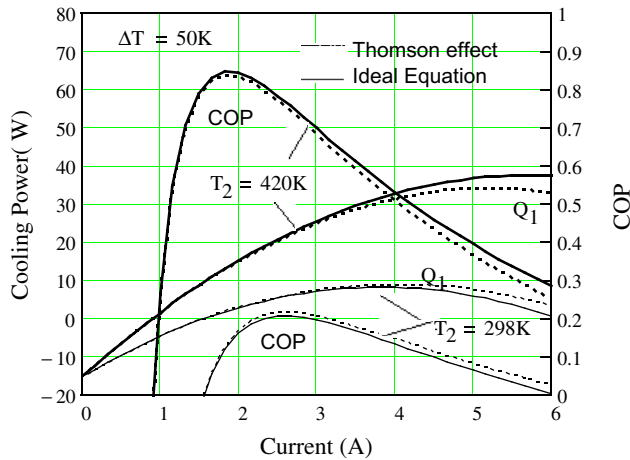


Fig. 7. Cooling power and COP vs. current at  $T_2 = 298$  K and  $T_2 = 420$  K for  $\Delta T = 50$  K  $\beta = -0.29$  at  $T_2 = 420$  K and  $\beta = 0.2$  at  $T_2 = 298$  K.

software ANSYS was deployed with the exactly same material properties, dimensions, and boundary conditions as those imposed on the exact solutions. Recall that the exact solutions were analytically obtained using mathematical software Mathcad.

The exact solutions with Equations (36) and (42) and the methodology used are justified by comparing them with the ANSYS simulations as shown in Fig. 9. The ANSYS solves numerically the governing equations of Equations (17), (20), (22), and (30) by means of the finite element method with the given boundary conditions [17]. It is found in Fig. 9 that the exact solutions are in precise agreement with the ANSYS simulations. Accordingly, the gaps between the exact solutions and the ideal equation implicate the quantities associated with the assumption made in the ideal equation. It is also deduced that the assumption of the negligible Thomson coefficient is indeed appropriate for either the moderate currents or the moderate temperature differences. However, the errors between the ideal equation and the exact solutions could reach up to 8% for the maximum current (here  $I = 3.9$  A) and the high temperature difference ( $\Delta T = 50$  K). All the procedures and

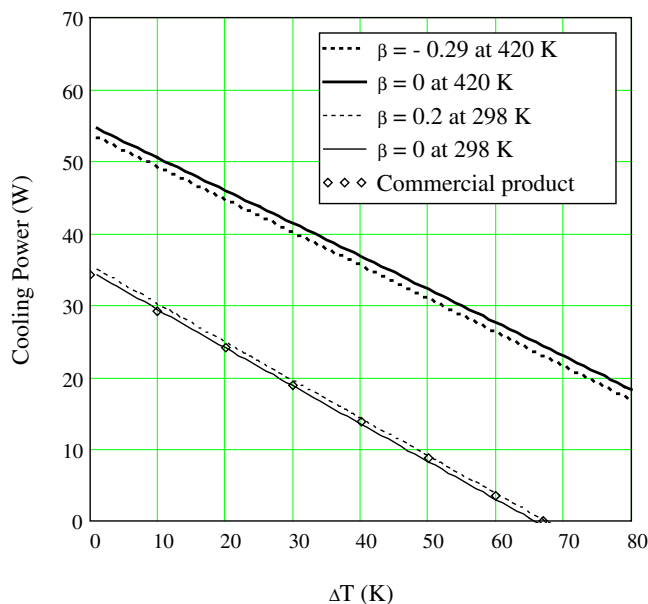


Fig. 8. Cooling power as a function of  $\Delta T$  at  $T_2 = 298$  K and 420 K for  $I = 3.9$  A.

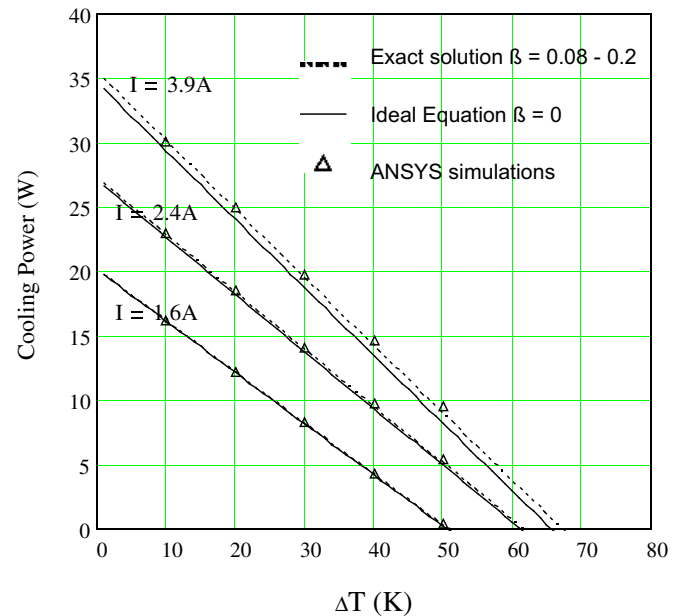


Fig. 9. Comparison of the exact solution with ANSYS simulations at  $T_2 = 298$  K.

conditions implemented in the exact solutions and the ideal equation turned out to be correct.

#### 4. Conclusions

A positive Thomson coefficient improves the performance of a thermoelectric cooler while a negative Thomson coefficient reduces the performance.

A substantial amount of heat absorbed by the Peltier cooling is wasted by the thermal conduction, substantially increasing with increasing temperature difference.

The exact solutions of the analytical differential equations including the Thomson effect are in excellent agreement with the ANSYS numerical simulations.

The ideal equation effectively predicts the performances of thermoelectric coolers if the Seebeck coefficient is correctly evaluated at the average temperature of the low and high junction temperatures.

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